

USING A TWO-AGE-CLASS LESLIE MODEL TO ANALYZE THE POPULATION GROWTH RATE OF Aedes Aegypti

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Abstract

Mosquitoes of the species *Aedes aegypti* are primary vectors of dengue, Zika, and chikungunya viruses, representing a major global health concern. Mathematical models of population dynamics provide valuable insights into vector control strategies. In this study, we apply a two-age-class Leslie matrix to investigate the growth rate of *Aedes aegypti* populations, partitioning the life cycle into juveniles (larvae) and reproductive adults. Survival and fertility parameters are derived from empirical entomological data. We mathematically derive the characteristic polynomial to determine the dominant eigenvalue and its corresponding eigenvector. Furthermore, we introduce sensitivity and elasticity analyses to highlight the relative impacts of larval survival and adult fecundity on the overall growth rate. The analytical results demonstrate that population expansion is strictly conditional on the product of survival and fertility rates. Under favorable ecological conditions, the dominant eigenvalue exceeds unity, indicating rapid expansion. However, under simulated vector control scenarios, falls below unity, confirming population decline. The mathematical derivations also prove that the stable age distribution is robust, relying on the square root of the survival-to-fertility ratio. The study provides a theoretical basis for combining fundamental matrix algebra with entomological data to identify highly targeted, actionable pathways for dengue prevention.

Keywords: *Aedes aegypti*, eigenvalue analysis, Leslie matrix, population dynamics, sensitivity analysis, vector control

1. Introduction

In population ecology, dynamics are frequently modeled using discrete-time frameworks to describe stage-structured evolution over time. The Leslie matrix model (Leslie, 1945) is a cornerstone of this approach, utilizing linear algebra to project age-structured populations. The fundamental theorem of Leslie matrices dictates that the asymptotic growth rate of a population is governed by the dominant eigenvalue (λ_1) of the projection matrix, while the corresponding right eigenvector establishes the stable age distribution (Caswell, 2001; Mahaffy, 2012).

The two-age-class Leslie model offers a highly tractable mathematical environment. With only two demographic stages—juveniles and adults—it captures the fundamental biological trade-offs: survival from the juvenile stage and adult reproduction. In the context of epidemiology, structured population models have been pivotal in forecasting disease vectors like *Aedes aegypti*, the primary carrier of dengue and Zika viruses (Bassani, 2023).

However, existing literature often jumps to complex, multi-stage or stochastic models, occasionally obscuring the direct analytical relationships between fundamental parameters. This study focuses strictly on the two-age-class Leslie matrix applied to *Aedes aegypti*. While utilizing a standard framework, the novelty of our approach lies in conducting a rigorous sensitivity and elasticity analysis on the dominant eigenvalue. This mathematical extension explicitly quantifies how variations in demographic traits (e.g., due to environmental shifts or vector control measures) mathematically force the dominant eigenvalue below unity, thereby halting population growth.

The paper is structured as follows: Section 2 formalizes the mathematical definition of the matrix and derives the analytical expressions for eigenvalues, eigenvectors, and elasticity. Section 3 applies this framework to empirical data across various ecological scenarios, culminating in an assessment of vector control strategies. Section 4 provides concluding remarks.

2. Materials and methods

2.1. The Two-Age-Class Leslie Matrix

We partition the *Aedes aegypti* population into two distinct classes:

- Class 1 (Larvae/Juveniles): Non-reproductive stage.
- Class 2 (Adult Females): Reproductive stage responsible for oviposition.

Let $n_t = \begin{bmatrix} N_1(t) \\ N_2(t) \end{bmatrix}$ be the population vector at time t where $N_1(t)$ and $N_2(t)$ denote the number of individuals (or population density) in Class 1 and Class 2, respectively. The projection from t to $t+1$ is governed by $n_{t+1} = Ln_t$, where L is the Leslie matrix:

$$L = \begin{bmatrix} f_1 & f_2 \\ s_1 & 0 \end{bmatrix}$$

Since larvae (Class 1) do not reproduce, $f_1 = 0$. Let $F = f_2$ denote the effective adult female fecundity (number of female offspring produced per adult female that survive to hatch), and $S = s_1$ denote the probability of a larva surviving to the adult stage. The matrix simplifies to:

$$L = \begin{bmatrix} 0 & F \\ S & 0 \end{bmatrix}$$

2.2. Eigenvalue, Eigenvector, and Elasticity Derivation

The asymptotic growth and age structure of this dynamical system $n_{t+1} = Ln_t$ are determined by the dominant eigenvalue (λ_1) and its corresponding eigenvector of L . The eigenvalues λ are the roots of the characteristic equation $\det(L - \lambda I) = 0$:

$$\det \begin{bmatrix} -\lambda & F \\ S & -\lambda \end{bmatrix} = \lambda^2 - FS = 0$$

The roots are $\lambda_{1,2} = \pm\sqrt{FS}$. The dominant eigenvalue, representing the long-term population growth rate, is therefore:

$$\lambda_1 = \sqrt{FS}$$

Biologically, the population grows exponentially if $\lambda_1 > 1$ (i.e., $FS > 1$), remains stable if $\lambda_1 = 1$, and declines towards extinction if $\lambda_1 < 1$.

The stable age distribution is given by the right eigenvector $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ corresponding to λ_1 , where v_1 and v_2 represent the relative proportions of larvae and adult females, respectively. Solving $(L - \lambda_1 I)v = 0$:

$$-\sqrt{FS}v_1 + Fv_2 = 0 \Rightarrow v_1 = \sqrt{\frac{F}{S}}v_2$$

Normalizing $v_2 = 1$, the eigenvector is $v = \begin{bmatrix} \sqrt{F/S} \\ 1 \end{bmatrix}$. Thus, the ratio of adults to larvae at equilibrium is mathematically defined as $\sqrt{S/F}$.

To evaluate the relative impact of proportional changes in F and S on population growth, we compute the elasticity, a standard metric in population dynamics, defined mathematically as $e_x = \frac{x}{\lambda_1} \frac{\partial \lambda_1}{\partial x}$.

For fecundity F :

$$e_F = \frac{F}{\sqrt{FS}} \left(\frac{1}{2} \sqrt{\frac{S}{F}} \right) = 0.5$$

For survival S :

$$e_S = \frac{S}{\sqrt{FS}} \left(\frac{1}{2} \sqrt{\frac{F}{S}} \right) = 0.5$$

This analytical result demonstrates that, in a standard two-stage framework without juvenile reproduction, the population growth rate is equally elastic to both adult fecundity and larval survival.

2.3. Empirical Background on *Aedes aegypti*

To parameterize the model, we rely on established entomological baselines. Female *Aedes aegypti* typically produce 50–150 eggs per gonotrophic cycle with a hatching rate of 60–80% under optimal conditions. Accounting for multiple cycles and a standard 1:1 sex ratio, the effective female fecundity (F) ranges from 10 to 30. Survival from larva to adult (S) is highly dependent on temperature and aquatic habitats, generally ranging from 0.2 to 0.4. Furthermore, high temperatures (e.g., above 30°C) or severe desiccation are known to drastically reduce both F and S .

Table 1 summarizes four distinct ecological scenarios based on these bounds, including a novel intervention scenario.

Table 1. Experimental parameter sets for the Leslie matrix

Scenario	F (Adult Fecundity)	S (Larval Survival)	Description
A	15	0.3	Normal conditions (25–28°C, moderate humidity)
B	25	0.4	Favorable conditions (optimal habitat availability)
C	10	0.2	Harsh conditions (drought, suboptimal temperature)
D	5	0.1	Intervention (active vector control/ larviciding)

Source: Parameter ranges are informed by entomological studies of *Aedes aegypti* (Arévalo-Cortés et al., 2022; Goindin et al., 2015; Đỗ et al., 2018) and population modeling frameworks (Leslie, 1945; Caswell, 2001; Dagmar, 2015; Bassani, 2023). Scenario D represents a hypothetical intervention case assumed for modeling purposes.

3. Results and Discussions

3.1. Growth Rate Analysis

Applying the derived formula $\lambda_1 = \sqrt{FS}$, we assess the population trajectories:

- **Scenario A:** $\lambda_1 = \sqrt{15 \times 0.3} = \sqrt{4.5} \approx 2.12$. The population more than doubles per projection interval.
- **Scenario B:** $\lambda_1 = \sqrt{25 \times 0.4} = \sqrt{10} \approx 3.16$. Optimal conditions trigger explosive outbreaks.
- **Scenario C:** $\lambda_1 = \sqrt{10 \times 0.2} = \sqrt{2} \approx 1.41$. Despite harsh environments, $FS > 1$, meaning the population still grows, albeit slower.
- **Scenario D:** $\lambda_1 = \sqrt{5 \times 0.1} = \sqrt{0.5} \approx 0.71$. Under vector control, $\lambda_1 < 1$. The population declines exponentially.

Contrary to the assumption that mosquito populations always grow naturally, our mathematical derivation explicitly proves that growth is strictly conditional on the threshold $FS > 1$. Scenario D illustrates that sufficient disruption to biological parameters reverses the growth trajectory.

3.2. Robustness of the Stable Age Distribution

Using the derived eigenvector ratio (Adults: Larvae = $1:\sqrt{F/S}$), we can analyze the structural balance of the population.

- **Scenario A:** Ratio = $\sqrt{0.3/15} \approx 0.141$ (approx. 12.4% adults, 87.6% larvae).
- **Scenario B:** Ratio = $\sqrt{0.4/25} \approx 0.126$ (approx. 11.2% adults, 88.8% larvae).
- **Scenario C:** Ratio = $\sqrt{0.2/10} \approx 0.141$ (approx. 12.4% adults, 87.6% larvae).
- **Scenario D:** Ratio = $\sqrt{0.1/5} \approx 0.141$ (approx. 12.4% adults, 87.6% larvae).

The mathematical derivation clarifies why the age distribution remains remarkably stable (roughly 11–13% adults) despite massive variances in λ_1 . Because F and S typically scale proportionally under environmental stress or even active vector control, the fraction $\sqrt{F/S}$ experiences minimal variance. This proves mathematically that measuring larval abundance is a consistently reliable proxy for adult density across different climatic conditions and intervention states.

3.3. Implications for Vector Control via Elasticity

The elasticity values ($e_F = 0.5$ and $e_S = 0.5$) derived in Section 2.2 provide profound insights for pest management. They dictate that a 10% reduction in larval survival (e.g., via larvicides or source reduction) has the exact same proportional impact on the population growth rate as a 10% reduction in adult fecundity (e.g., via Wolbachia infections or sterile insect techniques).

Consequently, integrated vector management (IVM) strategies that simultaneously target both stages (Scenario D) are mathematically optimal. By creating a multiplicative disruption beneath the square root ($\lambda_1 = \sqrt{FS}$), concurrent interventions force a rapid and compound collapse of the dominant eigenvalue, driving the population toward extinction more efficiently than isolated, single-stage tactics.

3.4. Implications for vector control

From a mathematical perspective, reducing either f_2 or s_1 directly lowers $\lambda_{\max}$, thereby slowing population growth. In practice:

- **Impact on the larval stage (reducing s_1):** Measures such as eliminating stagnant water sources (source reduction) or using larvicides can significantly reduce the survival rate from larvae to adult mosquitoes. This is a common strategy in vector control because the larval stage is vulnerable to environmental factors.
- **Impact on the reproductive capacity of adult female mosquitoes (reducing f_2):** Methods such as sterile male mosquito techniques, infecting with Wolbachia bacteria to reduce disease transmission and reproduction, or limiting the blood-feeding ability

of female mosquitoes can all reduce reproductive efficiency. This directly reduces the number of new mosquito generations produced.

Thus, the Leslie matrix is not only a mathematical tool for quantifying population growth rates, but also serves as a strategic analytical framework, helping to identify the most important biological parameters to influence. Combining mathematical analysis with practical measures allows for the development of effective vector control programs, contributing to reducing the risk of spreading diseases caused by *Aedes aegypti*.

4. Conclusion

This study utilizes a rigorous eigenvalue analysis of a two-age-class Leslie matrix to investigate *Aedes aegypti* population dynamics. By explicitly deriving the characteristic polynomial and eigenvectors, we clarified the underlying mechanics of population expansion and structural equilibrium. We demonstrated that population growth is not an absolute given but is strictly bounded by the condition $\lambda_1 = \sqrt{FS} > 1$. Furthermore, our elasticity analysis provides mathematical proof that larval survival and adult fecundity exert equal proportional leverage on the growth rate. These findings validate the necessity of multi-pronged vector control campaigns, offering a robust quantitative foundation for evaluating epidemiological interventions.

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